Wage Offers and On-the-job Search

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Abstract

We study the wage-setting problem of an employer with private information about demand for its product when workers can engage in costly on-the-job search. Employers understand that low wage offers may convey bad news that induces workers to search. The unique perfect sequential equilibrium wage strategy is characterized by: (i) pooling by intermediate-revenue employers on a common wage that just deters search; (ii) discontinuously lower revealing offers by low-revenue employers for whom the benefit of deterring search fails to warrant the required high pooling wage; and (iii) high revealing offers by high-revenue employers seeking to deter aggressive raiders.

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1 Introduction

Consider a company that experiences a large negative demand shock for its product. If its production entails the use of an input such as widgets, the company may want to open its books to its widget supplier to reveal this information. Even though opening its books is costly, revealing the bad news to its widget supplier may allow it to negotiate a lower price for its widgets that more than offsets those costs.

Employees are also inputs to production—but workers are not widgets. Revealing bad news may give workers incentives to search for new jobs, increasing the risk that the company loses valued employees who have acquired valuable skills. That is, a worker who learns that her company is in trouble may initiate costly on-the-job search to try to find a more secure job. If the worker leaves, the company's hardship will be exacerbated. To forestall such search, a company may work to conceal bad news from its workers, understanding that workers may search, but widgets do not.

This paper studies how the wage that an employer offers its workers varies with its information about demand for its product in such scenarios. We consider an employer that privately observes the market conditions underlying the revenues that a worker would generate. When the employer makes its wage offer, it considers the consequences for a worker's actions and probabilities of retention. In our simple framework, a worker's search motive is driven by a fear that her employer may be facing weak demand, and thus likely to shut down, leaving her unemployed. Because a worker only sees the wage offer and not the underlying market conditions, she uses the information contained in the wage offer to decide whether to engage in costly on-the-job search that improves the likelihood of drawing an outside offer. We identify and exhaustively characterize the unique perfect sequential equilibrium.

A worker's search decision depends on her employer's perceived viability: Bad news portending shutdown increases the value of soliciting outside offers, while good news reduces the value of such effort. An employer's equilibrium wage offer reflects an understanding of such considerations by its workers: In normal times—when expected revenues are neither very low nor very high—the employer chooses a common wage that does not vary with its expected revenues. This common wage is just high enough that it is only justifiable for an employer in reasonable health, just convincing the worker that times are not *so* bad that search is warranted. When times are tougher, setting this common wage to deter search ceases to be justifiable. Instead, an employer expecting low revenues opts for a discontinuously lower wage, understanding that the worker will search for outside offers upon observing the wage. When times are sufficiently good, by contrast, the employer offers more than the common wage to signal the good news and to reduce the risk of losing the worker who, despite not searching, may still be hired away by an aggressive outside firm.

Qualitatively, the keys for this characterization are only that (i) wage offers convey information about market conditions to workers; and (ii) higher wage offers raise the likelihood of retaining a worker. Thus, the results extend to alternative formulations of wage competition between the employer and raiding firms. For example, successful search may result in the worker being able to draw a wage from an exogenous distribution, or an employer may be able to sometimes, but not always, enter a bidding war and make a counter-offer in what becomes a second-price auction. The qualitative implications for wage setting extend beyond search to settings in which workers may take other actions that employers care about, provided that good news encourages the preferred action and employers with good news gain more from this action. For example, our analysis can characterize wage offers when a worker must decide whether to make a human capital investment in the presence of production complementarities between privately-observed demand and the investment.

The informational asymmetry between employers and workers has direct implications for job loss among workers. For employers that set the common wage but expect relatively low revenues, doing so serves to shroud unfavorable information about future viability, distorting their workers' search decisions: Upon observing the common wage, these workers incorrectly infer that their employers are likely in better health than they actually are, and hence do not search. Had the workers known the truth—had there been no informational asymmetry—they would have searched in order to insulate themselves from job loss. The result is avoidable job loss among workers for those employers that end up shuttering their doors. We study the welfare implications of employers' wage-setting decisions, comparing how total expected employer plus employee surplus under incomplete information relative to the complete-information benchmark varies with the employer's market demand and model primitives.

The premise that employers have private information about demand, and thus the likelihood of shutdown, is motivated by the Worker Adjustment and Retraining Notification (WARN) laws in the United States. The WARN Act of 1988 requires employers to provide workers with two-months advance notice of anticipated plant closings and mass layoffs.[1] Per

[1] WARN laws apply to employers with over 100 employees, excluding those who have worked fewer than six of the last 12 months and those who average fewer than 20 hours of work per week.

the Department of Labor, advance notice is intended to:

> *"...give workers and their families some transition time to adjust to the prospective loss of employment, to seek and obtain other jobs..."*

Such laws represent prima facie evidence of the informational structure and mechanism that we study—that employers have private information about their future viability and, in the absence of legislation mandating disclosure, will choose to withhold such information, thereby exposing their workers to avoidable job loss. That such disclosure laws are costly to employers (Clinebell and Clinebell (1994)) adds empirical weight to this argument.

The role of wages in mediating this informational asymmetry is motivated by an empirical literature documenting evidence that wages change relatively infrequently. While our framework is static, the pooling of wages across types that arises in equilibrium lends itself naturally to a dynamic interpretation: In a dynamic setting, following a change in market conditions, firms choose between pooling on a wage—typically the existing wage—versus increasing or decreasing the wage. In our static setting, we endogenize the pooling wage via our equilibrium refinement. While a fully dynamic model is beyond our paper's scope, our static notion of pooling thus naturally translates to a dynamic notion of "wage rigidity." Separately, our model is motivated by an empirical literature documenting cross-sectional evidence that, within firms, there is little variation in pay between different workers in similar jobs, and a more recent literature that exploits confidential administrative data to show that earnings growth distributions are negatively skewed and exhibit substantial kurtosis—all observations that our mechanism could plausibly help to explain.

Theoretically, our model is related to efficiency wage models, especially those with labor turnover motives (e.g., Stiglitz (1985)). A key difference is the form of informational asymmetry: In efficiency-wage models, wage-setting decisions reflect that a worker's action is private information to the worker and thus non-contractible, motivating a role for wages to influence worker behavior. Here, by contrast, wage-setting decisions reflect that *employers* have private information, motivating a role for wages to convey information to workers and affect search decisions. Our model is also related to Weingarden (2017), who studies employment decisions of firms in an asymmetric information setting in which firms seek to induce worker effort.

2 Model

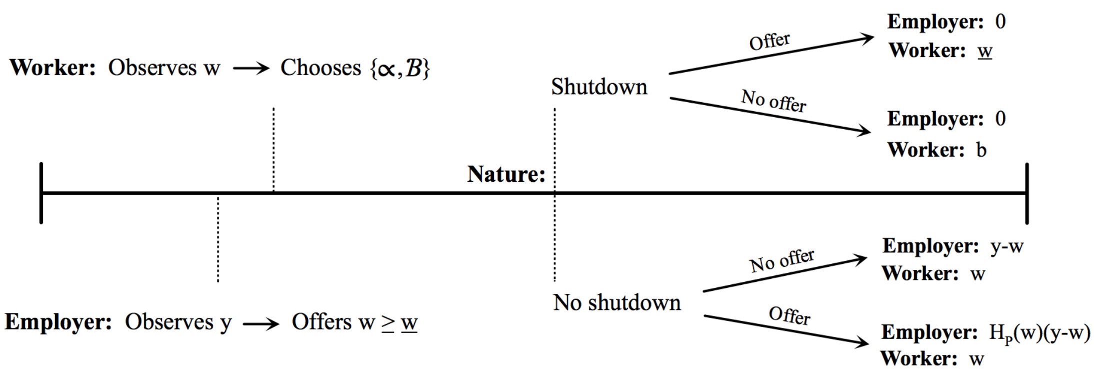
We study a one-period game between a risk-neutral worker and employer. The employer sees the market conditions that underlie the revenues a worker would generate and makes a wage offer. The worker only sees the wage offer and not the market conditions. Given the wage offer, the worker decides whether to engage in costly on-the-job search.

At the outset, the employer observes a signal $y \in [\underline{y}, \bar{y}] \equiv Y$ equal to the expected revenues that the worker will generate if she stays and the employer remains viable. We assume that y is drawn from a commonly-known distribution H_I that is twice continuously differentiable with density h_I. This revenue signal also contains information about the employer's future viability: with probability $g(y)$ the employer remains viable, but with probability $1 - g(y)$ the employer will shut down, lay off the worker and earn zero profits. We assume that $g'(y) > 0$: employers expecting higher revenues are more likely to be viable. After observing y, but prior to the shut-down shock, the employer makes a wage offer $w \geq \underline{w}$ that the worker would receive were she to stay with her employer who remains viable, where $\underline{w} \leq \underline{y}$ is a wage floor.[2] A laid-off worker who fails to find employment elsewhere receives unemployment benefit b, where $\underline{w} > b$. It eases presentation to assume that $\underline{y} = \underline{w}$.

The worker does not see y, but the employer's wage offer w conveys information about y. Given w, the worker decides whether to engage in costly on-the-job search. Search entails a fixed cost $\kappa > 0$, but improves the worker's likelihood of drawing an outside offer: a searching worker encounters a raiding firm with probability $\alpha \in (0, 1]$, while a non-searching worker does so with lower probability $\beta \in (0, \alpha)$. Here, $\beta > 0$ captures the two-sided nature of search—Faberman et al. (2017) find that employed job seekers receive high numbers of both solicited and unsolicited job offers, indicating that outside firms often initiate searches by directly contacting workers at other employers. We assume that the revenues the worker would generate at the raider are drawn independently from a distribution H_P with non-increasing density h_P on $[\underline{y}, \bar{y}]$ that could differ from H_I. The raider observes the employer's wage offer (and whether it survived) before making its wage offer. We also assume that the layoff risk is high enough that a worker would want to search were she to observe the lowest revenue signal $\underline{y}$: $(1 - g(\underline{y}))(\underline{w} - b)(\alpha - \beta) > \kappa$. These assumptions imply a strictly positive employment rent for the worker, i.e., for avoiding unemployment, giving rise to a precautionary motive for on-the-job search.

[2]Employers with $y < \underline{w}$ can never earn profits by retaining workers.

Figure 1: Timing of events



To ease analysis, the model is designed so the sole reason for search is this precautionary motive. To see this, note that if an incumbent employer survives, a worker who receives an outside offer joins the raider if and only if it offers a wage $w' \geq w$. If the employer shuts down, the worker joins the raiding firm if and only if it offers a feasible wage $w' \geq \underline{w}$. Thus, if the incumbent employer survives, a raider with $y' > w$ will hire the worker at wage $w' = w$, while a raiding firm with $y' \leq w$ will either not make an offer, or will offer $w' < w$, and have its offer rejected. If the incumbent shuts down, a raider hires the worker at $w' = \underline{w} > b$.

The model highlights the twin roles played by wage offers: (i) they convey information to workers about market conditions faced by employers, and hence the value of search; and (ii) when raiders seek to hire workers, higher wage offers increase the likelihood that employers retain workers. Our qualitative findings generalize when these features are preserved. Appendix C illustrates this for a more general notion of outside wage competition.

Complete information benchmark. As a prelude, we characterize equilibrium outcomes when a worker sees y before making search decisions. The employer's strategy is a wage offer function $\bar{\omega}(y)$ that maps each value of y into a wage offer w. Abusing notation slightly, the worker's strategy is a function $\bar{\sigma}(w, y)$ mapping each (w, y) pair into a search intensity $\{\alpha, \beta\}$, corresponding to the probability of receiving a wage offer from a raider. An equilibrium is a pair $\bar{\sigma}^*(w, y)$ and $\bar{\omega}^*(y)$ such that:

1. The worker searches, i.e. $\bar{\sigma}^*(w, y) = \alpha$, if and only if

$$\mathbb{E}\Big[(1-g(y))\Big((1-\alpha)b+\alpha\underline{w}\Big)+g(y)w-\kappa\Big|w,y\Big] > \mathbb{E}\Big[(1-g(y))\Big((1-\beta)b+\beta\underline{w}\Big)+g(y)w\Big|w,y\Big] \tag{1}$$

2. The wage offer $\bar{\omega}^*(y)$ maximizes expected profits given $\bar{\sigma}^*(w, y)$, solving

$$\max_{w \geq \underline{w}} \Big\{ g(y)(y-w)[1-\bar{\sigma}^*(w,y)(1-H_P(w))] \Big\}. \tag{2}$$

The definition of equilibrium subsumes the formalization of optimization by a raiding firm and a worker's choice of where to work, detailed above.

If a worker observes y, her optimization problem is simple. Low values of y imply a high probability of job loss, and thus a high precautionary value of on-the-job search. The solution to (1) implies that the worker's search decision is characterized by a cutoff y^* such that the worker searches if and only if $y < y^*$, where y^* solves $g(y^*) = 1 - \frac{\kappa}{(\alpha-\beta)(\underline{w}-b)}$. That is, the search decision only depends on y (save at y^*), and not the wage w, so we can abuse notation and write it as $\bar{\sigma}^*(y) = \alpha$ if $y < y^*$ and $\bar{\sigma}^*(y) = \beta$ if $y > y^*$.

The employer's optimization problem is also simple. Expected employer profits are strictly concave in w. Thus, when interior, the employer's wage offer is given implicitly by the unique solution $w(y, \bar{\sigma}^*(y))$ to the first-order condition for (2),

$$(y-w)\bar{\sigma}^*(y)h_P(w) - [1-\bar{\sigma}^*(y)(1-H_P(w))] = 0, \tag{3}$$

and it equals $\underline{w}$ otherwise. That is, the employer's equilibrium wage offer is given by $\bar{\omega}^*(y) = \underline{w}$ for $y < \hat{y}(\bar{\sigma}^*(y))$, and $\bar{\omega}^*(y) = w(y, \bar{\sigma}^*(y))$ for $y \geq \hat{y}(\bar{\sigma}^*(y))$, where $\hat{y}(\bar{\sigma}^*(y))$ solves

$$(\hat{y}-\underline{w})(1-\bar{\sigma}^*(\hat{y})) = (\hat{y}-w(\hat{y},\bar{\sigma}^*(\hat{y})))(1-\bar{\sigma}^*(\hat{y})(1-H_P(w(\hat{y},\bar{\sigma}^*(\hat{y}))))).$$

Inspection of (3) reveals that, fixing $\bar{\sigma}^*(y)$, the marginal payoffs of higher wages rise with y, reflecting that employers with better prospects lose more if workers leave. So, too, fixing y, the marginal payoffs of higher wages rise with the probability that workers receive outside offers.

When expected revenues from retaining workers are very low, employers offers the lowest feasible wage that will retain workers who fail to receive an outside wage offer. When expected revenues are high enough, an employer competes by offering a higher wage that trades off the reduced profits when the worker is retained against the increased likelihood of retaining a worker who encounters a raider.[3]

[3]We assume that $\bar{y} > \hat{y}(\beta)$, i.e., the risk that an employer fails can be sufficiently low relative to the cost κ of search that workers do not search.

Incomplete Information. We next characterize equilibrium outcomes when workers do not see y prior to deciding whether to search. In this environment, workers make inferences about y, and hence the likelihood of layoff $1 - g(y)$, based on the wage offers they receive.

The employer's strategy is a function $\omega(y)$ mapping each y into a wage offer w. The worker's strategy is a function $\sigma(w)$ mapping each wage offer w into a search intensity $\{\alpha, \beta\}$. A worker belief function is a function mapping each feasible wage offer w into a probability distribution $\mu(y|w)$ over y.

Definition. *A pure-strategy perfect Bayesian equilibrium (PBE) is a pair of strategies, $\sigma^*(w)$ and $\omega^*(y)$, and a worker belief function $\mu(y|w)$ such that:*

1. *The worker engages in costly search, i.e. $\sigma^*(w) = \alpha$, if*

$$\mathbb{E}_\mu\Big[(1-g(y))\Big((1-\alpha)b+\alpha\underline{w}\Big)+g(y)w-\kappa\Big|w\Big] > \mathbb{E}_\mu\Big[(1-g(y))\Big((1-\beta)b+\beta\underline{w}\Big)+g(y)w\Big|w\Big], \tag{4}$$

   but not if the inequality is reversed.

2. *The employer's wage offer $\omega^*(y)$ maximizes expected profits given $\sigma^*(w)$, solving*

$$\max_{w \geq \underline{w}} \Big\{g(y)(y-w)(1-\sigma^*(w)(1-H_P(w))\Big\}. \tag{5}$$

3. *For all possible equilibrium-path wages (i.e. $\forall w \in \{w|\exists y \in Y \text{ with } \omega^*(y) = w\}$), beliefs $\mu(y|w)$ are updated via Bayes' rule.*

Definition. *A pure-strategy perfect sequential equilibrium (PSE) is any pure-strategy PBE for which beliefs following out-of-equilibrium wage offers satisfy the credibility condition:*

For all wages not occurring in equilibrium (i.e. $\forall \tilde{w} \notin \{w|\exists y \in Y \text{ with } \omega^*(y) = w\}$), there does not exist a corresponding set of revenue types $J(\tilde{w})$ such that

$$\mathbb{E}[\pi(y, \omega^*(y), \sigma^*(\omega^*(y)))] < \min_{\tilde{\sigma} \in BR(J(\tilde{w}), \tilde{w})} \Big\{\mathbb{E}[\pi(y, \tilde{w}, \tilde{\sigma})]\Big\} \quad \Longleftrightarrow \quad y \in J(\tilde{w}). \tag{6}$$

We focus on pure-strategy PSE, i.e., equilibria that satisfy the credibility condition of Grossman and Perry (1986). After characterizing PSE, we describe the properties possessed by all PBE. Appendix A exhaustively characterizes all PBE.

Workers search when the offered wage indicates a sufficiently high probability of job loss. A worker's search decision is again characterized by a cutoff rule, but now the cutoff rule reflects a worker's beliefs about $g(y)$ following the wage offer w: $\sigma^*(w) = \alpha$ if $\mathbb{E}_\mu[g(y)|w] < g(y^*)$, and $\sigma^*(w) = \beta$ if $\mathbb{E}_\mu[g(y)|w] > g(y^*)$.

An employer's optimization problem reflects the dual role played by wage offers: wage offers directly affect the likelihood of retaining a worker *and* they convey information about market conditions, potentially affecting search decisions. Proposition 1 establishes that the equilibrium is unique given a restriction to credible beliefs.[4,5]

Proposition 1. *There is an essentially unique PSE wage strategy $\omega^*(\cdot)$. The strategy is characterized by:*

$\mathcal{W}1$ *A unique pooling wage, w^p, given by $\underline{w}$ if $\mathbb{E}_\mu[g(y)|\underline{w}] > g(y^*)$, and otherwise solving $\mathbb{E}_\mu[g(y)|w^p] = g(y^*)$. The pooling region Y^p is bounded from below by $\underline{y}$ if $\mathbb{E}[\pi(\underline{y}, w^p, \beta)] > \mathbb{E}[\pi(y, \bar{\omega}^*(\underline{y}, \alpha), \alpha)]$, and otherwise by the y solving $\mathbb{E}[\pi(y, w^p, \beta)] = \mathbb{E}[\pi(y, \bar{\omega}^*(y, \alpha), \alpha)]$. It is bounded from above by $\bar{y}$ if $\bar{y} < \bar{\omega}^{*-1}(w^p, \beta)$, and otherwise by $\bar{\omega}^{*-1}(w^p, \beta)$.*

When $\underline{y}$ is sufficiently low (i.e., $\underline{y} < \inf\{y|\mathbb{E}[\pi(y, w^p, \beta)] = \mathbb{E}[\pi(y, \bar{\omega}^*(y, \alpha), \alpha)]\}$) and $\bar{y}$ is sufficiently high (i.e., $\bar{y} > \bar{\omega}^{*-1}(w^p, \beta)$), then:

$\mathcal{W}2$ *Lower types $y < \inf\{Y^p\}$ offer low revealing (complete-information) wages that lead their workers to search.*

$\mathcal{W}3$ *Type $y = \inf\{Y^p\}$ is indifferent between setting w^p and the discontinuously lower complete-information wage that induces search.*

$\mathcal{W}4$ *Types $y \in Y^p$ pool on a wage just high enough to deter search: $\mathbb{E}_\mu[g(y)|w^p] = g(y^*)$.*

$\mathcal{W}5$ *Higher types $y > \max\{Y^p\}$ offer high revealing (complete-information) wages that strictly discourage search.*

$\mathcal{W}6$ *w^p is the complete-information wage of the highest pooling type.*

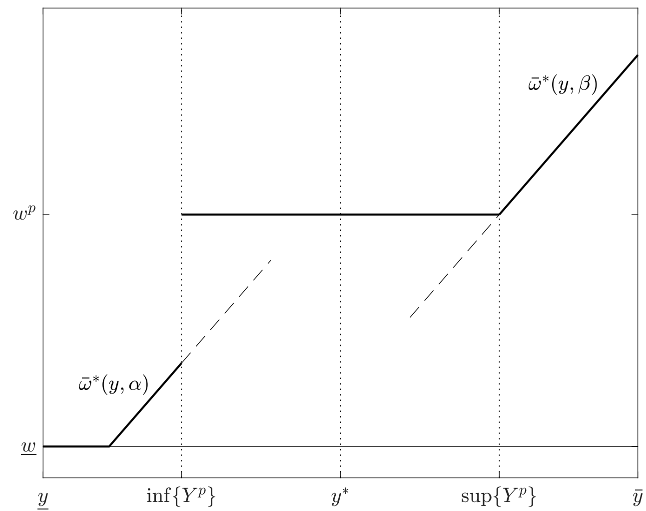
Proof. See Appendix B. □

That employers anticipating intermediate revenues pool on a common wage and thus conceal their private information reflects the informational role of wage offers. Employers receiving positive signals about revenues place greater value on the employment relationship, benefiting more from retaining employees. As a result, $\omega^*(\cdot)$ weakly increases with an

[4] The set of PSE is unique up to indifference by the revenue type who either reveals and induces search or pools and deters search.

[5] We let $\bar{\omega}^*(y, \sigma)$ denote the optimal complete-information wage for fixed search decision σ. To ease notation, we write $\bar{\omega}^{*-1}(\underline{w}, \sigma) = \max\{y|\bar{\omega}^*(y, \sigma) = \underline{w}\}$.

Figure 2: Incomplete information (unique PSE)



Notes: Uniform uncertainty with $g(y) = y$.

employer's expected revenue. Therefore, higher wage offers signal higher expected revenues, reducing a worker's incentive to search. It follows that, among employers for whom revealing induces search, those expecting relatively high revenues offer a high wage—above what they would offer absent informational asymmetries—to emulate a higher type whose worker would prefer not to search. Pooling thus emerges in any PSE.

The precise structure and uniqueness of the pooling wage offer in $\mathcal{W}1$ reflects the requirement that workers' beliefs be credible. To understand, observe that if employers pool on some wage $w^p > \underline{w}$ for which $\mathbb{E}_\mu[g(y)|w^p] > g(y^*)$, then there is a slightly lower wage, $w^p - \epsilon$, that is preferred to w^p by a coalition including (i) non-pooling types $y < \inf\{Y^p\}$ for whom $w^p - \epsilon$ would rationalize deviating from their low revealing wage in order to deter search, and (ii) pooling types for whom $w^p - \epsilon$ is closer to their complete-information wage, provided that such a deviation deters search. Because this coalition consists only of relatively high types, such deviations must induce workers to believe $\mathbb{E}_\mu[g(y)|w^p - \epsilon] > g(y^*)$, to which their unique best response is not to search. Thus, for any such $w^p > \underline{w}$, there always exists a profitable deviation to $w^p - \epsilon$ when beliefs must be credible.

$\mathcal{W}2$–$\mathcal{W}6$ characterize the equilibrium wage structure $\omega^*(y)$ when the distribution of revenue types has sufficient support. Figure 2 depicts the key qualitative features.

$\mathcal{W}2$ establishes that as long as the prospects for some employers are sufficiently dire, i.e., as long as $\underline{y}$ is sufficiently low, then some employers do not value the employment relationship enough to justify paying the common pooling wage needed to deter search. Such employers understand that lower wage offers will reveal that economic conditions are so bad that workers should search; thus, they do best to offer their optimal complete-information wage. Employers expecting higher revenues value the employment relationship by more—the opportunity cost of losing a worker is higher. $\mathcal{W}3$ establishes that some employer is just indifferent between offering a low wage that reveals low expected revenues and induces search, and offering the discontinuously higher pooling wage that induces beliefs that expected revenues exceed what they in fact are, deterring search.

Employers with better outlooks than this indifferent type thus prefer to offer wages that deter search. $\mathcal{W}4$ establishes that the pooling wage leaves workers just indifferent between searching and not: $\mathbb{E}_{\mu}[g(y)|w^p] = g(y^*)$. The sufficient support assumption ensures that $w^p > \underline{w}$. Were a higher pooling wage offered such that $\mathbb{E}_{\mu}[g(y)|w^p] > g(y^*)$, then a coalition of relatively high types would deviate to a wage slightly below w^p, as described above. In response, workers with credible beliefs would forego search knowing that only relatively high types would consider such a deviation, so such wage strategies cannot be part of a PSE.

Employers with even better prospects whose complete-information wages exceed w^p know that such offers will signal high expected revenues and thus also deter search. For such employers, pooling on w^p requires making an uncompetitively low offer (given their high valuations of workers) while conferring no benefits in terms of search behavior. Hence, they offer their complete-information wages. As $\mathcal{W}5$ details, provided $\bar{y}$ is sufficiently high, wages eventually rise smoothly from the pooling wage, reflecting that employers expecting such high revenues want to offer a wage over and above the pooling wage to reduce the risk of losing workers to raiding firms. Finally, $\mathcal{W}6$ establishes that employers who deter search offer the pooling wage if it weakly exceeds the wage they would offer if they could ensure that a worker would not search.

The equilibrium structure of wages described above speaks to several well-documented empirical features of wage distributions. First, a large literature documents that wages only change infrequently, with downward revisions occurring particularly rarely (e.g. Altonji and Devereux (2000), Gottschalk (2005), etc.). While our framework is static, our results naturally admit a more dynamic interpretation: Instead of describing the firm's decision of whether or not to change the wage from some pre-existing level in response to changes in market conditions, we provide a characterization of the (unique) wage on which firms pool that arises endogenously in equilibrium. Allowing for repeated transitory shocks to a firm's expected revenue in our model delivers a dynamic notion of "wage rigidity" that bears directly on such evidence. Second, a distinct literature documents cross-sectional evidence of intra-firm wage compression. Groshen (1991) documents that over 90% of wage variation is explained by occupation and establishment alone: In industries with relatively few incentive workers, worker-level variation can explain only 3-7% of total variation in wages (see also Baker, Gibbs and Holmstrom (1994)). This complementary evidence is consistent with the characterization of wages that arises in our model. Third, a more recent empirical literature (e.g. Karahan et al. (2017)) exploits confidential employer-employee matched administrative panel data on earnings to establish that earnings growth distributions among job stayers are negatively skewed and exhibit substantial kurtosis. In our model, the discontinuous drop in wages among employers who cannot rationalize offering the pooling wage to deter search suggests a mechanism that can help account for these observations.

Finally, the characterization of wages that arises from the informational asymmetry in our model is not innocuous in terms of employment outcomes. Employers offering the pooling wage for whom expected revenues are relatively low deter search by workers who would have searched in the absence of informational asymmetries. Because search increases the likelihood of obtaining outside offers, such workers fail to effectively insulate themselves from job loss. The measure

$$(\alpha - \beta)\Big[H_I(g(y^*)) - H_I(\inf\{Y^p(w^p)\})\Big] > 0 \tag{7}$$

of workers become unemployed due to the informational asymmetry.

Wage competition. The qualitative properties of the equilibrium extend when competition between incumbent and raider is enriched to capture additional real world features of labor markets. For example, they extend when successful search yields a draw with positive probability from some non-trivial wage offer distribution. Such a scenario would arise when successful search yields contact with a single raider who either does not observe the employer's offer, or does not know the worker's valuation of her current job because the worker cannot credibly reveal its non-pecuniary value. Even when an incumbent can sometimes make a counter-offer and thus compete with potential raiders in the auction, the incentive to set a wage above that required to deter search in order to preemptively outbid outside offers is reduced but the underlying logic is unchanged. We establish this formally in Appendix C.

Such scenarios increase workers' incentives to search given any wage and belief about her employer due to the added benefit of increasing the probability of securing a higher outside offer. An employer's wage offer now affects search decisions via two distinct channels: the information channel emphasized in our model (through which a higher wage signals better news, reducing search incentives), and a direct channel via its level (through which higher offers reduce the likelihood of dominating outside offers, likewise reducing search incentives). Because employers expecting higher revenues lose more when workers leave, such considerations raise their incentives to deter search.[6]

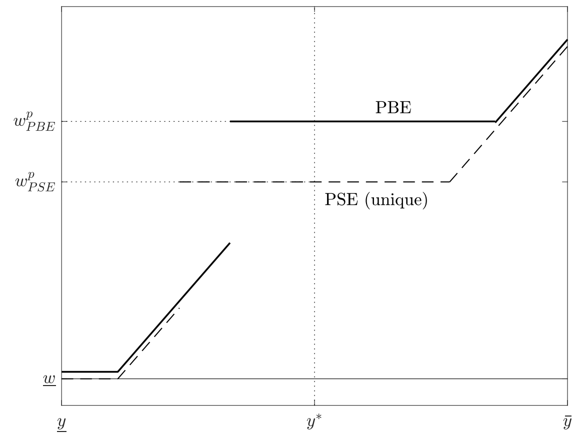
Because both wage channels affect search in the same direction, incorporating such additional features would result in higher equilibrium pooling wages, but would not otherwise alter the model's qualitative properties. Indeed, adding these two assumptions, one can drop the precautionary motive for search and assume $g(y) = 1$ for all y.

Perfect Bayesian equilibria. When beliefs are not required to be credible following unexpected wage offers, multiple equilibria emerge. Nonetheless, Appendix A shows that *all* PBE possess many properties of the unique PSE: (a) wage offers weakly increase with y, reflecting that the opportunity cost of losing workers rises with y; (b) employers expecting low revenues offer low wages—equal to those made when demand is public information—thereby revealing bad news and inducing search, reflecting that the employment relationship is not sufficiently valuable to justify the higher wage needed to deter search; (c) employers expecting intermediate revenues pool on the lowest wage that deters search, as higher search-deterring wages would entail over-bidding with no countervailing benefit in terms of search behavior; and (d) employers expecting high revenues who reveal their types (except possibly $\bar{y}$) offer wages arbitrarily close to their complete-information optima—wages that diverge non-trivially would induce emulation by similar types who would gain from offering wages closer to their complete-information optimum (so the offer would not be revealing).

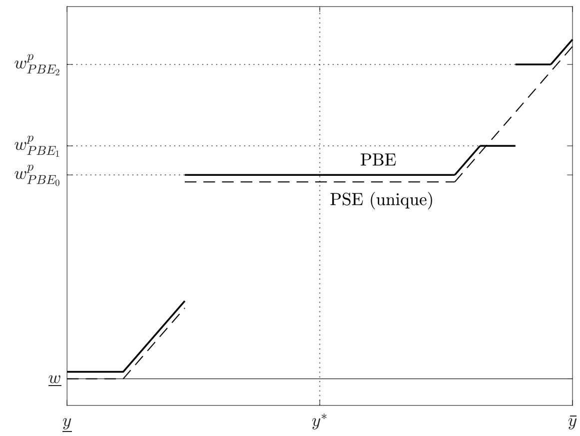
The multiplicity of PBE reflects two possibilities that are sustained only by incredible beliefs. One possibility, depicted in Figure 3(a), is equilibria with pooling on wages above the pooling wage in Proposition 1. Such equilibria result if pooling on otherwise preferred lower wages would induce pessimistic worker beliefs and hence search. If workers can adopt such pessimistic beliefs, pooling on the higher wage is sustained. But such pessimism is not credible following small deviations from a high pooling wage: Only employers expecting high revenue value the employment relationship enough to offer such wages. The second possibility, depicted in Figure 3(b), is equilibria featuring multiple pooling regions at higher

[6] A similar rationale emerges in the preemptive-wage setting of Scoones and Bernhardt (1998).

Figure 3: Incomplete information (multiple PBE)



(a) High pooling wage



(b) Multiple pooling wages

Notes: Uniform uncertainty with $g(y) = y$.

wages. Such equilibria are sustained by a similar argument applied to employers expecting higher revenue: Workers observing high off-path wages can adopt an incredible degree of pessimism to credibly threaten to search, rendering deviations away from high pooling wages unprofitable for high types. The result is a proliferation of PBE, each featuring (possibly many) connected segments of high-revenue type employers pooling on high wages that may be above or below the wages they would optimally choose conditional on deterring search.

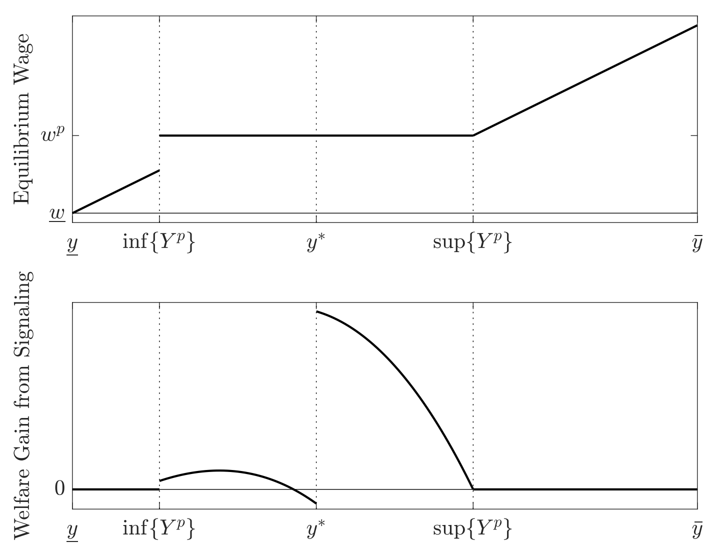
Welfare. We next investigate the welfare implications of the wage-setting behavior of employers and the search decisions of workers. We measure welfare as the expected total surplus accruing to the employer and worker—i.e., the employer's expected profits plus the worker's expected compensation—and study how the welfare differences between the signaling (i.e., from informational incompleteness) and complete-information benchmarks vary with y.

There are three principal externalities governing the welfare gain (or loss) associated with signaling. First, the employer does not internalize the wage gain that a worker receives when it offers a higher wage and remains viable but the worker is, nonetheless, poached away. Second, the employer does not internalize the negative effect of its wage offer, and the induced search decision, on the likelihood of costly unemployment in the event of a shutdown: The measure $(\alpha - \beta)\Big[H_I(g(y^*)) - H_I(\inf\{Y^p(w^p)\})\Big] > 0$ of workers who decide to not search in the incomplete-information environment, but who otherwise would have searched, face an increased risk of unemployment, and thus lost income, if the incumbent fails to remain viable.

Third, workers do not internalize the profit losses incurred by employers when they quit.

The net effect of these three forces, and thus the welfare effect of incomplete information, depends on the level of demand y. For very low and very high types ($y < \inf\{Y^p\}$ and $y > \sup\{Y^p\}$), the optimal wage strategy under incomplete information coincides with the complete-information wage, so there is no welfare difference. For moderately high types ($y \in (y^*, \sup\{Y^p\})$), workers prefer not to search regardless of the information structure. It follows that the second and third externalities discussed above—in particular, those associated with distorted search decisions—disappear. The only remaining externality is the first, associated with the worker attracting better outside offers when its current employer offers a higher wage. Because such employers cannot set a lower wage without inducing the worker to search when information is incomplete, the relatively high pooling wage just represents a redistribution from employer to worker in the event that there is no separation, and hence nets out in terms of total surplus; but the worker gains when she is poached, so total welfare necessarily rises in expectation. For moderately low types ($y \in (\inf\{Y^p\}, y^*)$), by contrast, the high pooling wage associated with informational incompleteness deters workers from searching when they otherwise would have preferred to search. Thus, in such cases, all three externalities are operative and the net effect on welfare is ambiguous.

Figure 4: Welfare



Notes: Uniform uncertainty with $g(y) = y$.

The top panel of Figure 4 depicts the unique PSE wage strategy associated with uniformly-distributed demand (H_P,$H_I \sim U[0,1]$) and a survival probability that is linear in demand ($g(y) = y$). The bottom panel depicts the associated welfare gain from signaling, reflecting the preceding intuition: Welfare is unaffected at the extremes, unambiguously higher when information is incomplete for moderately high types, and potentially higher or lower, depending on which of the externalities dominates, for moderately low types.

3 Conclusion

The observation at the heart of this paper—that when employers have private information, wage offers convey news that affects workers' decisions—is germane to many settings. So long as workers are inclined to take the action preferred by their employers—be it abstaining from on-the-job search, investing in firm-specific human capital, exerting extra effort, etc.—when they believe their employer is in good health, and so long as that action is more valuable to employers in better health, the qualitative structure of wage offers should obtain. In particular, equilibrium wages are characterized by: pooling by intermediate employer types on a common wage that just encourages the worker to take the desired action; discretely lower revealing offers by low-revenue employers for whom the value of the desired action does not warrant the higher pooling wage needed to induce the action; and high revealing offers by high-revenue employers for whom workers would happily take the desired action, so that other considerations dictate wage offers. Such generality suggests that the mechanism we identify has wide-ranging empirical relevance for wage-setting that merits further investigation.

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Appendix

To conserve notation, let $r(w,\sigma) \equiv 1-\sigma(1-H_P(w))$ denote the probability that the employer retains a worker given w and σ. Slightly abusing notation, let $\bar{\omega}^*(y,\alpha)$ and $\bar{\omega}^*(y,\beta)$ denote the complete-information optimal wage offers given search and no-search, respectively. Finally, let $Y^s \equiv \{y|\sigma^*(\omega^*(y)) = \alpha\}$ and $Y^{ns} \equiv \{y|\sigma^*(\omega^*(y)) = \beta\}$ denote the set of y for which the employer's wage offer induces search and no-search, respectively.

A. Characterization of PBE

Propositions 2 and 3 characterize the set of pure-strategy PBE wage strategies.

Proposition 2. *The equilibrium wage strategy $\omega^*(\cdot)$ has the following properties:*

W1 Wage offers are weakly increasing in revenues: If $y < y'$, then $\omega^*(y) \leq \omega^*(y')$.

W2 If a wage offer induces a worker to search, then it equals the complete information wage: If $y \in Y^s$, then $\omega^*(y) = \bar{\omega}^*(y)$.

W3 Only low revenue employers make wage offers that induce workers to search: If $y \in Y^s$ and $y' \in Y^{ns}$, then $y' > y$.

W4 Employers expecting sufficiently high revenues make wage offers that discourage search: Y^{ns} is non-empty.

W5 Employer profits, $\mathbb{E}[\pi(y,\omega^*(y),\sigma^*(\omega^*(y)))]$, are continuous in y.

Proposition 3. *Equilibrium wage offers that deter search have the following properties:*

W6 The set of revenue types that offer a pooling wage that deters search is connected: If $y < y' \in Y^{ns}$ and $\omega^*(y') = \omega^*(y)$, then $\omega^*(y'') = \omega^*(y)$, $\forall\ y'' \in (y,y')$.

W7 If $\omega^*(y)$ discourages search and is not part of a pooling wage offer, i.e. if $\nexists\ y'$ such that $\omega^*(y') = \omega^*(y)$, then either it is arbitrarily close to the complete information wage, i.e., for all $\epsilon > 0$, $|\omega^*(y) - \bar{\omega}^*(y)| < \epsilon$, or the employer is the highest revenue type $\bar{y}$.

W8 If Y^s and Y^{ns} are non-empty, then wages jump at the highest y for which the worker searches: $\omega^*(y)$ is discontinuous upward at $\sup\{Y^s\}$.

W9 There is pooling at the lowest wage that deters search: $\exists y' > \inf\{Y^{ns}\}$ *such that* $\forall y \in (\inf\{Y^{ns}\}, y')$, $\omega^*(y) = w^p$. *Furthermore,* $y' \geq \min\{\bar{y}, \sup\{y|\bar{\omega}^*(y,\beta) = w^p\}\}$.

W10 If $\mathbb{E}[g(y)] < g(y^*)$, *then* Y^s *is non-empty.*

Proof. We consider Propositions 2 and 3 together.

W1 If $y < y'$, *then* $\omega^*(y) \leq \omega^*(y')$.

If the search decision is the same after $\omega^*(y)$ as $\omega^*(y')$ then it follows immediately that $\omega^*(y) \leq \omega^*(y')$ because for a given probability σ of an outside offer, $\frac{\partial^2 \mathbb{E}[\pi(y,w,\sigma)]/g(y)}{\partial y \partial w} = \sigma h_P(w) > 0$, for $w > \underline{y}$. Moreover, the optimal complete information wage increases in the probability that the worker gets an outside offer, precluding $\omega^*(y) > \omega^*(y')$ for $y \in Y^{ns}$, $y' \in Y^s$. It remains to rule out $w \equiv \omega^*(y) > \omega^*(y') \equiv w'$ for $y \in Y^s$, $y' \in Y^{ns}$. This is immediate if $r(w,\alpha) \leq r(w',\beta)$. Suppose $r(w,\alpha) > r(w',\beta)$. Optimization by y requires

$$\begin{aligned} & \mathbb{E}[\pi(y,w,\alpha)] \geq \mathbb{E}[\pi(y,w',\beta)] \\ \Longrightarrow\ & y[r(w,\alpha) - r(w',\beta)] \geq wr(w,\alpha) - w'r(w',\beta) \\ \Longrightarrow\ & y'[r(w,\alpha) - r(w',\beta)] > wr(w,\alpha) - w'r(w',\beta) \\ \Longrightarrow\ & \mathbb{E}[\pi(y',w,\alpha)] > \mathbb{E}[\pi(y',w',\beta)] \end{aligned}$$

contradicting optimization by y' types.

W2 If $y \in Y^s$, *then* $\omega^*(y) = \bar{\omega}^*(y)$.

Immediate. For any $w \neq \bar{\omega}^*(y)$, we have $\mathbb{E}[\pi(y,w,\alpha)] < \mathbb{E}[\pi(y,\bar{\omega}^*(y),\alpha)] < \mathbb{E}[\pi(y,\bar{\omega}^*(y),\beta)]$. The first inequality follows from optimality of $\bar{\omega}^*(y)$ given search and the second from the fact that deterring search raises profits for a given wage. Thus, y types prefer $\bar{\omega}^*(y)$ to w.

W3 If $y \in Y^s$ *and* $y' \in Y^{ns}$, *then* $y' > y$.

Suppose $y' < y$. By W1, $\omega^*(y') \leq \omega^*(y)$. Because $y' \in Y^{ns}$ and $y \in Y^s$, it must be that $\omega^*(y') < \omega^*(y)$. But then, because $\sigma^*(\omega^*(y')) = \beta$ by assumption, if search decisions are monotonically decreasing in w, it must also be that $\sigma^*(\omega^*(y)) = \beta$, a contradiction. To see that search decisions are monotonically decreasing in w, note that in the model in the text, wages only affect search through beliefs. Thus, because W1 together with Bayes' rule imply that $\mathbb{E}_\mu[g(y)|w]$ is increasing in w, it must be that search is decreasing in w.[7]

[7]More generally, if we wish to consider an alternative model of poaching, for the claim to hold it is sufficient to show that the expected gain from searching is weakly decreasing in w given beliefs about $g(y)$.

W4 Y^{ns} is non-empty.

If Y^{ns} is empty, then $\omega^*(y) = \bar{\omega}^*(y)$ for all y and wage offers are revealing. But then when offered $\bar{\omega}^*(\bar{y})$, beliefs are such that $\mathbb{E}_\mu[g(y)|\bar{\omega}^*(\bar{y})] = g(\bar{y}) > g(y^*)$, so a worker prefers to not search.

W5 $\mathbb{E}[\pi(y, \omega^*(y), \sigma^*(\omega^*(y)))]$ is continuous in y.

Suppose $\mathbb{E}[\pi(y, \omega^*(y), \sigma^*(\omega^*(y)))]$ is discontinuous at $y_0 \in Y$. Then $\exists \epsilon > 0$ such that for any $\delta > 0$, $\exists y'$ with $|y' - y_0| < \delta$ but $|\mathbb{E}[\pi(y', \omega^*(y'), \sigma^*(\omega^*(y')))] - \mathbb{E}[\pi(y_0, \omega^*(y_0), \sigma^*(\omega^*(y_0)))]| > \epsilon$. Continuity of $\mathbb{E}[\pi(y, \tilde{w}, \sigma^*(\tilde{w}))]$ for fixed $\tilde{w}$ implies that $\forall \epsilon' > 0$, $\exists \delta' > 0$ such that if $|y - y_0| < \delta'$, then $|\mathbb{E}[\pi(y, \tilde{w}, \sigma^*(\tilde{w}))] - \mathbb{E}[\pi(y_0, \tilde{w}, \sigma^*(\tilde{w}))]| < \epsilon'$. Set $\epsilon' = \epsilon$ and $\delta = \delta'$ and let $w' \equiv \omega^*(y')$ and $w_0 \equiv \omega^*(y_0)$. Then y' satisfies the preceding and

$$|\mathbb{E}[\pi(y', \tilde{w}, \sigma^*(\tilde{w}))] - \mathbb{E}[\pi(y_0, \tilde{w}, \sigma^*(\tilde{w}))]| < \epsilon < |\mathbb{E}[\pi(y', w', \sigma^*(w')] - \mathbb{E}[\pi(y_0, w_0, \sigma^*(w_0))]|.$$

If $y' > y_0$, then setting $\tilde{w} = w'$ implies $\mathbb{E}[\pi(y_0, w_0, \sigma^*(w_0))] < \mathbb{E}[\pi(y_0, w', \sigma^*(w'))]$, so w_0 is not optimal for y_0 (since $\mathbb{E}[\pi(y, \omega^*(y), \sigma^*(\omega^*(y)))]$ increases in y). If $y' < y_0$, then setting $\tilde{w} = w_0$ implies $\mathbb{E}[\pi(y', w_0, \sigma^*(w_0))] > \mathbb{E}[\pi(y', w', \sigma^*(w'))]$, so w' is not optimal for y'.

W6 Let y, $y' \in Y^{ns}$. If $y < y'$ and $\omega^*(y') = \omega^*(y)$, then $\omega^*(y'') = \omega^*(y)$, $\forall y'' \in (y, y')$.

Suppose $\exists y'' \in (y, y')$ such that $\omega^*(y'') \neq \omega^*(y)$. Let $w \equiv \omega^*(y) = \omega^*(y')$, $w'' \equiv \omega^*(y'')$ and $y'' = cy + (1-c)y'$ for some $c \in (0, 1)$. $\omega^*(\cdot)$ is weakly increasing for all types in Y^{ns} ($\frac{\partial^2 \mathbb{E}[\pi(y,w,\beta)]/g(y)}{\partial y \partial w} = \beta h_P(w) > 0$), so $y'' \in Y^s$. But then optimality of w for y, y' implies

$$cy[r(w, \beta) - r(w'', \alpha)] \geq c[wr(w, \beta) - w''r(w'', \alpha)]$$
$$(1-c)y'[r(w, \beta) - r(w'', \alpha)] > (1-c)[wr(w, \beta) - w''r(w'', \alpha)].$$

Adding yields

$$y''[r(w, \beta) - r(w'', \alpha)] > wr(w, \beta) - w''r(w'', \alpha),$$

contradicting optimality of w'' for y''.

To see this, let $V^{s-ns}(w, \mathbb{E}_\mu[g(y)|w])$ denote the expected gain from searching. Then for any $w < w'$ we have

$$V^{s-ns}(w, \mathbb{E}_\mu[g(y)|w]) \geq V^{s-ns}(w', \mathbb{E}_\mu[g(y)|w]) > V^{s-ns}(w', \mathbb{E}_\mu[g(y)|w'])$$
$$\implies V^{s-ns}(w, \mathbb{E}_\mu[g(y)|w]) > V^{s-ns}(w', \mathbb{E}_\mu[g(y)|w'])$$

where the first inequality follows from the the condition that the expected gain from searching is weakly decreasing in w for given beliefs, and the second inequality follows from W1 and Bayes' rule.

W7 Let $y \in Y^{ns}$. If $\nexists\ y'$ such that $\omega^*(y') = \omega^*(y)$, then either $|\omega^*(y) - \bar{\omega}^*(y)| < \epsilon$ for all $\epsilon > 0$, or $y = \bar{y}$.

Suppose not. First note that $y \neq \underline{y}$, or else the worker infers that $y = \underline{y} < y^*$, and hence searches. But if she searches, it is optimal to offer $\bar{\omega}^*(\underline{y})$. So consider $y \in (\underline{y}, \bar{y})$.

Because $\mathbb{E}[\pi(y, w, \beta)]$ is single-peaked and continuous in w, if $\omega^*(y) < \bar{\omega}^*(y)$, then $\sigma^*(\omega^*(y)) = \beta$ and $\nexists w' \in (\omega^*(y), \bar{\omega}^*(y)]$ such that $\sigma^*(w') = \beta$. Similarly, if $\omega^*(y) > \bar{\omega}^*(y)$, then $\sigma^*(\omega^*(y)) = \beta$ and $\nexists w' \in [\bar{\omega}^*(y), \omega^*(y))$ such that $\sigma^*(w') = \beta$.

Suppose now that $\bar{\omega}^*(y) - \sup\{w|w < \bar{\omega}^*(y) \text{ and } \sigma^*(w) = \beta\} > 0$. Then, because marginal profits of a higher w increase in y for fixed σ and $\mathbb{E}[\pi(y, w, \beta)]$ is continuous in y, there exists a $\delta > 0$ with $y - \delta > \underline{y}$ such that for all $y' \in (y - \delta, y)$, we have $\omega^*(y') = \max\{w|w < \bar{\omega}^*(y) \text{ and } \sigma^*(w) = \beta\} = \omega^*(y)$, a contradiction of the premise. Thus, if $\omega^*(y) < \bar{\omega}^*(y)$, then $\bar{\omega}^*(y) = \sup\{w|w < \bar{\omega}^*(y) \text{ and } \sigma^*(w) = \beta\}$. An analogous argument precludes $\omega^*(y) > \bar{\omega}^*(y)$ for $y < \bar{y}$.

W8 If Y^s and Y^{ns} are non-empty, then $\omega^*(y)$ is discontinuous at $\sup\{Y^s\}$.

Immediate. Fixing the probability that a worker receives an offer, employer profits are continuous in w, but higher by $g(y)(y - w)(\alpha - \beta)H_P(w)$ if the worker does not search for a fixed w. The result then follows from optimization in the neighborhood of $\sup\{Y^s\}$.

W9 $\exists y' > \inf\{Y^{ns}\}$ such that $\forall y \in Y^{ns}$ with $y \leq y'$, $\omega^*(y) = w^p$. Furthermore, $y' \geq \min\{\bar{y}, \max\{y|\bar{\omega}^*(y, \beta) = w^p\}\}$.

Suppose there is no $y' \in Y^{ns}$ such that all $y \in Y^{ns}$ with $y \leq y'$ pool on some w^p. If Y^s is non-empty, then $\inf\{\omega^*(y)|y \in Y^{ns}\} < \sup\{\omega^*(y)|y \in Y^s\}$, contradicting monotonicity. If Y^s is empty, then $\omega^*(\underline{y})$ is revealing and induces search, contradicting the premise.

Furthermore, it must be that $y' \geq \min\{\bar{y}, \max\{y|\bar{\omega}^*(y, \beta) = w^p\}\}$, or else there will be a type $y'' > y'$ for which $\omega^*(y'') > w^p \geq \bar{\omega}^*(y'', \beta)$, contradicting optimality of $\omega^*(y'')$ for y''.

W10 If $\mathbb{E}[g(y)] < g(y^*)$, then Y^s is non-empty.

From monotonicity of $\omega^*(y)$ in y, $\mathbb{E}_\mu[g(y)|\omega^*(\underline{y})] \leq \mathbb{E}[g(y)]$. Thus, $\mathbb{E}_\mu[g(y)|\omega^*(\underline{y})] < g(y^*)$, so the worker prefers to search following $\omega^*(\underline{y})$.

□

B. Characterization of PSE

Proof.

$\mathcal{W}1$ *There is an essentially unique PSE wage strategy* $\omega^*(\cdot)$. *The strategy is characterized by a unique pooling wage,* w^p, *given by* $\underline{w}$ *if* $\mathbb{E}_\mu[g(y)|\underline{w}] > g(y^*)$, *and otherwise solving* $\mathbb{E}_\mu[g(y)|w^p] = g(y^*)$. *The pooling region* Y^p *is bounded from below by* $\underline{y}$ *if* $\mathbb{E}[\pi(\underline{y}, w^p, \beta)] > \mathbb{E}[\pi(y, \bar{\omega}^*(\underline{y}, \alpha), \alpha)]$, *and otherwise by the* y *solving* $\mathbb{E}[\pi(y, w^p, \beta)] = \mathbb{E}[\pi(y, \bar{\omega}^*(y, \alpha), \alpha)]$. *It is bounded from above by* $\bar{y}$ *if* $\bar{y} < \bar{\omega}^{*-1}(w^p, \beta)$, *and otherwise by* $\bar{\omega}^{*-1}(w^p, \beta)$.

We first establish uniqueness of w^p. By W9, at least one pooling wage, w^p, exists. Suppose there are multiple, and consider type $\tilde{y}$ pooling on $w^{p'} > w^p$ with $w^{p'} \neq \bar{\omega}^*(\tilde{y})$ and $g(\tilde{y}) > \mathbb{E}_\mu[g(y)|w^{p'}]$. Then $\tilde{w} \equiv \bar{\omega}^*(\tilde{y}) > w^p$ and $g(y^*) \leq \mathbb{E}_\mu[g(y)|w^p] < \mathbb{E}_\mu[g(y)|w^{p'}] < g(\tilde{y})$. Note that $\tilde{w}$ is not offered in equilibrium: if it were, then $\sigma^*(\tilde{w}) = \beta$, so $\tilde{y}$ should deviate from $w^{p'}$ to $\tilde{w}$.

To show that the credibility condition is violated for $\tilde{w}$, define $J(\tilde{w}) \equiv \{y|\mathbb{E}[\pi(y, \tilde{w}, \beta)] > \mathbb{E}[\pi(y, \omega^*(y), \sigma^*(\omega^*(y)))]\}$. Clearly, $\tilde{y} \in J(\tilde{w})$, so $J(\tilde{w})$ is non-empty. Furthermore, for all $y \in Y^s$, $\mathbb{E}[\pi(y, \bar{\omega}^*(y), \alpha)] \geq \mathbb{E}[\pi(y, w^p, \beta)] > \mathbb{E}[\pi(y, \tilde{w}, \beta)]$ from optimization by $y \in Y^s$ and single-peakedness, so $Y^s \cap J(\tilde{w}) = \emptyset$.

In fact, by continuity of profits in y (W5), $w^p > \bar{\omega}^*(\inf\{y|\omega^*(y) = w^p\}, \alpha) > \bar{\omega}^*(\inf\{y|\omega^*(y) = w^p\}, \beta)$, so sufficiently low types that pool on w^p strictly prefer that to deviating to $\tilde{w}$, regardless of whether $\tilde{w}$ deters search. Therefore $\mathbb{E}_\mu[g(y)|y \in J(\tilde{w})] > g(y^*)$, so the worker should not search following $\tilde{w}$. Thus, the credibility condition fails.

We next establish that w^p is given by $\underline{w}$ if $\mathbb{E}_\mu[g(y)|\underline{w}] > g(y^*)$ and otherwise solves $\mathbb{E}_\mu[g(y)|w^p] = g(y^*)$. Suppose not. Then $w^p > \underline{w}$ and $\mathbb{E}_\mu[g(y)|w^p] > g(y^*)$. To show that the credibility condition is violated, consider some $\tilde{\tilde{w}} < w^p$, and define $J'(\tilde{\tilde{w}}) \equiv \{y|\mathbb{E}[\pi(y, \tilde{\tilde{w}}, \beta)] \geq \mathbb{E}[\pi(y, \omega^*(y), \sigma^*(\omega^*(y)))]\}$. $\mathbb{E}_\mu[g(y)|y \in J'(\tilde{\tilde{w}})]$ is weakly increasing and continuous in $\tilde{\tilde{w}}$. Thus, for any $\epsilon > 0$, $\exists \delta > 0$ such that for all $\tilde{\tilde{w}} \in (w^p - \delta, w^p)$, we have $\mathbb{E}_\mu[g(y)|y \in J'(w^p)] - \mathbb{E}_\mu[g(y)|y \in J'(\tilde{\tilde{w}})] < \epsilon$. Set $\epsilon \equiv \mathbb{E}_\mu[g(y)|y \in J'(w^p)] - g(y^*)$ and note that $\mathbb{E}_\mu[g(y)|y \in J'(w^p)] = \mathbb{E}_\mu[g(y)|w^p] > g(y^*)$. Therefore, $\mathbb{E}_\mu[g(y)|y \in J'(\tilde{\tilde{w}})] > g(y^*)$. For $\tilde{\tilde{w}} < w^p$, the measure of types that strictly prefer $\tilde{\tilde{w}}$ to $\omega^*(y)$ equals the measure of types that weakly prefer $\tilde{\tilde{w}}$ to $\omega^*(y)$, so $\mathbb{E}_\mu[g(y)|y \in J(\tilde{\tilde{w}})] = \mathbb{E}_\mu[g(y)|y \in J'(\tilde{\tilde{w}})] > g(y^*)$, violating the credibility condition.

Finally, we establish the bounds on $Y^p(w^p)$. Consider the lower bound, $\inf\{Y^p\}$. If

$\mathbb{E}[\pi(\inf\{Y^p\}, w^p, \beta)] > \mathbb{E}[\pi(\inf\{Y^p\}, \bar{\omega}^*(\inf\{Y^p\}, \alpha), \alpha)]$, then either $\inf\{Y^p\} = \underline{y}$ or by continuity $\exists y < \inf\{Y^p\}$ with $\mathbb{E}[\pi(y, w^p, \beta)] > \mathbb{E}[\pi(y, \bar{\omega}^*(y, \alpha), \alpha)]$, in which case $\bar{\omega}^*(y)$ is not optimal for y. Conversely, if $\mathbb{E}[\pi(\inf\{Y^p\}, w^p, \beta)] < \mathbb{E}[\pi(\inf\{Y^p\}, \bar{\omega}^*(\inf\{Y^p\}, \alpha), \alpha)]$, then by continuity $\exists y \in Y^p$ with $\mathbb{E}[\pi(y, w^p, \beta)] < \mathbb{E}[\pi(y, \bar{\omega}^*(y, \alpha), \alpha)] \leq \mathbb{E}[\pi(y, \bar{\omega}^*(y, \alpha), \beta)]$, in which case w^p is not optimal for y.

Consider the upper bound, $\max\{Y^p\}$. If $\max\{Y^p\} < \bar{\omega}^{*-1}(w^p, \beta)$ then either $\max\{Y^p\} = \bar{y}$ or W9 is violated. If $\max\{Y^p\} > \bar{\omega}^{*-1}(w^p, \beta)$ then, by single-peakedness, all pooling types $y > \bar{\omega}^{*-1}(w^p, \beta)$ prefer $\bar{\omega}^*(y, \beta)$ to w^p because $\sigma^*(\bar{\omega}^*(y, \beta)) = \beta$, contradicting optimality of w^p for these types.

When $\underline{y}$ is sufficiently low (i.e., $\underline{y} < \inf\{y | \mathbb{E}[\pi(y, w^p, \beta)] = \mathbb{E}[\pi(y, \bar{\omega}^*(y, \alpha), \alpha)]\}$) and $\bar{y}$ is sufficiently high (i.e., $\bar{y} > \bar{\omega}^{*-1}(w^p, \beta)$), then:

$\mathcal{W}2$ *Lower types $y < \inf\{Y^p\}$ offer low revealing (complete-information) wages that induce workers to search.*

All low types $y < \inf\{Y^p\}$ induce search (W3) and all such types set $\bar{\omega}^*(y, \alpha) < w^p$ (W2).

$\mathcal{W}3$ *Type $y = \inf\{Y^p\}$ is indifferent between setting w^p and the discontinuously lower complete-information wage that induces search.*

The claim follows immediately from $\mathcal{W}1$ and the sufficient support restriction.

$\mathcal{W}4$ *Types $y \in Y^p$ pool on a wage w^p just high enough to deter search, i.e., $\mathbb{E}_\mu[g(y)|w^p] = g(y^*)$.*

$\mathbb{E}_\mu[g(y)|y \in Y^p(w^p)]$ is monotone increasing in w^p, so $w^p > \underline{w}$ uniquely solves $\mathbb{E}_\mu[g(y)|w^p] = g(y^*)$ provided $\mathbb{E}_\mu[g(y)|\underline{w}] < g(y^*)$. But if $\mathbb{E}_\mu[g(y)|\underline{w}] \geq g(y^*)$, then $\inf\{Y^p\} = \underline{y}$, a contradiction.

$\mathcal{W}5$ *Higher types $y > \max\{Y^p\}$ offer high revealing (complete-information) wages that strictly discourage search.*

All high types $y > \max\{Y^p\}$ discourage search (W3). All such types who don't pool set $\bar{\omega}^*(y, \beta)$ by single-peakedness, as no other type prefers $\bar{\omega}^*(y, \beta)$, and so $\sigma^*(\bar{\omega}^*(y, \beta)) = \beta$.

$\mathcal{W}6$ *w^p is the complete-information wage of the highest pooling type.*

The claim follows immediately from $\mathcal{W}1$ and the sufficient support restriction.

□

C. Alternative models of wage competition

Our main qualitative characterizations extend to various alternative models of wage competition. To see this, note that if the incumbent is committed to its initial offer, the following conditions are sufficient for the proofs of Propositions 1-3 to hold:

1. The expected gain from searching is weakly decreasing in w, given beliefs.
2. The benefit of a higher wage is greater for higher types.
3. Expected profits are single-peaked in w.

Thus, to determine whether our qualitative results generalize for any particular alternative model of wage competition, it suffices to verify that these conditions hold.

Consider, for example, a scenario in which successful search allows the worker to draw a wage w' from some exogenous distribution H_P. Such a scenario would arise, for example, if the raiding firm could not observe the incumbent's offer, or if the current job has some non-pecuniary value that the worker cannot credibly reveal to the raiding firm. To see that Condition 1 is satisfied, first note that the expected payoff from successful search is given by $\mathbb{E}[\max\{x, w'\}]$ where $w' \sim H_P$ and $x \in \{b, w\}$ are constants. Then, the expected gain from searching with wage offer w in hand may be written as

$$(\alpha - \beta)\left[\Big(1 - \mathbb{E}_\mu[g(y)|w]\Big)\big[\mathbb{E}[\max\{b, w'\}] - b\big] + \mathbb{E}_\mu[g(y)|w]\big[\mathbb{E}[\max\{w, w'\}] - w\big]\right] - \kappa.$$

For fixed beliefs $\mathbb{E}_\mu[g(y)|w]$, the expected gain from searching with wage offer w in hand is thus weakly decreasing in w so long as $\frac{\partial \mathbb{E}[\max\{w, w'\}]}{\partial w} = H_P(w) \leq 1$. Condition 1 is therefore satisfied. To see that Conditions 2 and 3 are satisfied, simply note that, for fixed σ, the firm's expected profit function is unchanged: $\mathbb{E}[\pi(y, w, \sigma)] = g(y)(y - w)[1 - \sigma(1 - H_P(w))]$. Condition 2 then follows immediately from the cross-partial: $\frac{\partial^2 \mathbb{E}[\pi(y, w, \sigma)]/g(y)}{\partial y \partial w} = \sigma h_P(w) > 0$. Condition 3 likewise follows immediately from strict concavity of expected profits in w.